THE BLOOD FLOWS RATE IN THE BLOOD VESSELS*

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Abstract

The blood flows in the blood vessels are considered. The velocities profile in the cylindrical blood vessels along the radial direction and axial direction are calculated. The equations of the blood flow rate and the blood pressure are determined by Navier-stokes equation and equation of continuity. This paper discusses a mathematical relation Hagen Poiseuille's equation about how much blood flow through an individual blood vessel and what would the rate of blood flow affect.

Keywords: velocity profile, the rate of blood flow, the blood pressure

Introduction

All the vessels are assumed to be the same in nature excluding their size, length and cross-sectional areas. The blood is the viscous fluid. The steady laminar flows of viscous incompressible fluid between two infinite stationary parallel plates are studied. Plane Couette flow is calculated that the lower plate is moving and the upper plate is at rest [6]. The steady laminar flow of without body forces for viscous incompressible fluid through an infinite circular pipe and two coaxial circular cylinders are discussed [8]. This flow is axi-symmetry and steady. The cardiovascular system equation is calculated by Navier-Stokes equation in the cylindrical coordinates [2]. The velocities of radial direction and axial direction in an axi-symmetry flow are considered [7].

In this paper, modeling of the blood flow rate and the flow rate of the blood flow in the blood vessel are produced by the cardiovascular system equation. Modeling of the blood pressure is calculated from the blood flow rate equation.

2. The governing equation

The velocity components in the r,θ and z directions are u,v and w respectively.

The Navier-Stokes equation in the cylindrical coordinates is given by

$$(1)\ \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\upsilon\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v}{\partial \theta}+\frac{\partial^2 u}{\partial z^2}\right],$$

$$(2)\ \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+\frac{uv}{r}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial \theta}+\upsilon\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv)\right)+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u}{\partial \theta}+\frac{\partial^2 v}{\partial z^2}\right],$$

$$(3)\ \frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \theta}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\upsilon\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial^2 w}{\partial z^2}\right],$$

where $\rho=$density, $\upsilon=\dfrac{\mu}{\rho}$ viscosity of fluid, and $p=$the fluid pressure.

The fluid is axi-symmetry with the assumption of no tangential velocity $(v=0)$ and the remaining quantities are independent of z. A change of variables on the Navier-Stokes equation in the cylindrical coordinates system:

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(4) $\dfrac{\partial w}{\partial t}+f\dfrac{\partial w}{\partial r}+w\dfrac{\partial w}{\partial z}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}+\upsilon\left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial w}{\partial r}\right)+\dfrac{\partial^2 w}{\partial z^2}\right],$

(5) $\dfrac{\partial f}{\partial t}+f\dfrac{\partial f}{\partial r}+w\dfrac{\partial f}{\partial z}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}+\upsilon\left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial f}{\partial r}\right)+\dfrac{\partial^2 f}{\partial z^2}-\dfrac{f}{r^2}\right],$

(6) $\dfrac{1}{r}\dfrac{\partial}{\partial r}(rf)+\dfrac{\partial w}{\partial z}=0.$

The equations (4) and (5) reduce to

(7) $\dfrac{\partial w}{\partial t}+f\dfrac{\partial w}{\partial r}+w\dfrac{\partial w}{\partial z}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}+\upsilon\left[\dfrac{\partial^2 w}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial w}{\partial r}+\dfrac{\partial^2 w}{\partial z^2}\right],$

(8) $\dfrac{\partial f}{\partial t}+f\dfrac{\partial f}{\partial r}+w\dfrac{\partial f}{\partial z}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}+\upsilon\left[\dfrac{\partial^2 f}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial f}{\partial r}+\dfrac{\partial^2 f}{\partial z^2}-\dfrac{f}{r^2}\right],$

where f be the radial flow component and w be the axial flow components in z - direction. The equation of continuity can be written as:

(9) $\dfrac{\partial \rho}{\partial t}+\dfrac{\partial(\rho w)}{\partial z}=0$.

Define a new variable which is the radial coordinate transformation, given by:

(10) $\gamma=\dfrac{r}{R(z,t)}$,

where $R(z,t)$ be the inner radius of the vessel, r is the radial direction.

By using equation (10) in equation (7),

(11) $$\frac{\partial w}{\partial t}+(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial R}\cdot\frac{\partial R}{\partial t})+f(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial r})+w(\frac{\partial w}{\partial z}+\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial R}\cdot\frac{\partial R}{\partial z})$$

$$=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left[(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial r})(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial r})+\frac{1}{r}(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial r})+(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial z})(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial z})\right].$$

Then Substituting $\dfrac{\partial \gamma}{\partial R}=\dfrac{\partial}{\partial R}(\dfrac{r}{R})$, $\dfrac{\partial \gamma}{\partial r}=\dfrac{\partial}{\partial r}(\dfrac{r}{R})$ and $\dfrac{\partial \gamma}{\partial z}=\dfrac{\partial}{\partial z}(\dfrac{r}{R})$ in equation (11),

$$\frac{\partial w}{\partial t}+\left(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial}{\partial R}(\frac{r}{R}).\frac{\partial R}{\partial t}\right)+f\left(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial}{\partial r}(\frac{r}{R})\right)+w\left(\frac{\partial w}{\partial z}\cdot\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial R}(\frac{r}{R})\cdot\frac{\partial R}{\partial z}\right)$$

$$=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left[\left(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial}{\partial r}(\frac{r}{R})\right)\left(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial}{\partial r}(\frac{r}{R})\right)+\frac{1}{r}\left(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial}{\partial r}(\frac{r}{R})\right)+\left(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial}{\partial z}(\frac{r}{R})\right)\left(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial}{\partial z}(\frac{r}{R})\right)\right] \quad (12)$$

$$\frac{\partial w}{\partial t}+\left(\frac{\partial w}{\partial \gamma}(\frac{-r}{R^2}).\frac{\partial R}{\partial t}\right)+f\left(\frac{\partial w}{\partial \gamma}(\frac{1}{R})\right)+w\left(\frac{\partial w}{\partial z}+\frac{\partial w}{\partial \gamma}(\frac{-r}{R^2})\frac{\partial R}{\partial z}\right)$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left[\left(\frac{\partial w}{\partial \gamma}(\frac{1}{R})\right)\left(\frac{\partial w}{\partial \gamma}(\frac{1}{R})\right) + \frac{1}{r}\left(\frac{\partial w}{\partial r}(\frac{1}{R})\right)\right].$$

By Factorizing and simplifying the left-hand side of equation (12),

(13) $$\frac{\partial w}{\partial t} - \frac{1}{R}\left[\gamma(\frac{\partial R}{\partial t} + \frac{\partial R}{\partial z}) - f\right]\frac{\partial w}{\partial \gamma} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho R^2}\left[\frac{\partial^2 w}{\partial \gamma^2} + \frac{1}{\gamma}\frac{\partial w}{\partial \gamma}\right].$$

From equation (6),

(14) $$\frac{\partial f}{\partial r} + \frac{f}{r} + \frac{\partial w}{\partial z} = 0.$$

By using the radial coordinate transformation equation (10),

$$\frac{\partial f}{\partial \gamma}\frac{\partial \gamma}{\partial r} + \frac{f}{r} + \left(\frac{\partial w}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial R}\cdot\frac{\partial R}{\partial z}\right) + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial f}{\partial \gamma}\frac{1}{R} + \frac{f}{r} - \frac{\partial w}{\partial \gamma}\frac{r}{R^2}\cdot\frac{\partial R}{\partial z} + \frac{\partial w}{\partial z} = 0.$$

By substituting $r = \gamma R$,

(15) $$\frac{\partial f}{\partial \gamma}\frac{1}{R} + \frac{f}{\gamma R} - \frac{\partial w}{\partial \gamma}\frac{r}{R^2}\cdot\frac{\partial R}{\partial z} + \frac{\partial w}{\partial z} = 0.$$

Multiplying equation (15) by γR,

$$\gamma\frac{\partial f}{\partial \gamma} + f - \gamma^2\frac{\partial w}{\partial \gamma}\cdot\frac{\partial R}{\partial z} + \gamma R\frac{\partial w}{\partial z} = 0.$$

Integrating the equation with respect to γ,

$$\int_0^\gamma \gamma\frac{\partial f}{\partial \gamma}d\gamma + \int_0^\gamma f d\gamma + \int_0^\gamma \gamma R\frac{\partial w}{\partial z}d\gamma - \int_0^\gamma \gamma^2\frac{\partial w}{\partial \gamma}\cdot\frac{\partial R}{\partial z}d\gamma = 0.$$

Since $w(0,\ z,\ t) = f(0,z,t) = 0$,

$$\gamma f(\gamma,z,t) + R\int_0^\gamma \gamma\frac{\partial w}{\partial z}d\gamma - \frac{\partial R}{\partial z}\gamma^2 w + \frac{\partial R}{\partial z}\int_0^\gamma 2w\gamma d\gamma = 0,$$

$$\gamma f(\gamma,z,t) = -R\int_0^\gamma \gamma\frac{\partial w}{\partial z}d\gamma + \frac{\partial R}{\partial z}\gamma^2 w - \frac{\partial R}{\partial z}\int_0^\gamma 2w\gamma d\gamma.$$

Dividing by γ,

(16) $$f(\gamma,z,t) = \gamma\frac{\partial R}{\partial z}w - \frac{R}{\gamma}\int_0^\gamma \gamma\frac{\partial w}{\partial z}d\gamma - \frac{2}{\gamma}\frac{\partial R}{\partial z}\int_0^\gamma w\gamma d\gamma,$$

where $f(\gamma)$ represents an arbitrary function satisfying

(*) $$\int_0^1 \gamma f(\gamma)d\gamma = 1.$$

Boundary conditions are

$$f\left(\gamma,z,t\right)\Big|_{\gamma=0}=0,\quad \frac{\partial w}{\partial \gamma}\left(\gamma,z,t\right)\Big|_{\gamma=0}=0,$$

$$f\left(\gamma,z,t\right)\Big|_{\gamma=1}=\frac{\partial R}{\partial t},\quad w\left(\gamma,z,t\right)\Big|_{\gamma=1}=0.$$

The function γ must satisfy the condition $f\left(1\right)=0$ since $\dfrac{\partial w}{\partial z}=0$ on $\gamma=1$.

Let $f\left(\gamma\right)$ be a power series in γ truncated to finite order N and imposing the conditions equation $(*)$ and $f\left(1\right)=0$.

(17) Let $f(\gamma)=\dfrac{2}{N}\displaystyle\sum_{k=1}^{N}\frac{k+1}{k}(\gamma^{2k}-1)$.

From equation (16),

$$-\int_0^1 \gamma\frac{\partial w}{\partial z}d\gamma=\int_0^1 \gamma f(\gamma)\frac{1}{R}\frac{\partial R}{\partial t}d\gamma+\frac{2}{R}\frac{\partial R}{\partial z}\int_0^1 \gamma w d\gamma\,,$$

$$-\int_0^1 \gamma\frac{\partial w}{\partial z}d\gamma=\int_0^1\left[\gamma f(\gamma)\frac{1}{R}\frac{\partial R}{\partial t}+\frac{2}{R}\frac{\partial R}{\partial z}\gamma w\right]d\gamma\,,$$

(18) $$-\int_0^1 \gamma\frac{\partial w}{\partial z}d\gamma=\int_0^1 \gamma\left[\frac{f(\gamma)}{R}\frac{\partial R}{\partial t}+\frac{2w}{R}\frac{\partial R}{\partial z}\right]d\gamma\,.$$

Taking approximation of considering the equality between the integral to integrands for equation (18),

(19) $$-\frac{\partial w}{\partial z}=\frac{f(\gamma)}{R}\frac{\partial R}{\partial t}+\frac{2w}{R}\frac{\partial R}{\partial z}\,.$$

Substituting the equation (17) into equation (19),

$$-\frac{\partial w}{\partial z}=\frac{2w}{R}\frac{\partial R}{\partial z}-\frac{1}{R}\frac{\partial R}{\partial t}\left[-\frac{2}{N}\sum_{k=1}^{N}\frac{k+1}{k}(\gamma^{2k}-1)\right],$$

(20) $$\frac{\partial w}{\partial z}=-\frac{2w}{R}\frac{\partial R}{\partial z}+\frac{1}{R}\frac{\partial R}{\partial t}\left[\frac{2}{N}\sum_{k=1}^{N}\frac{k+1}{k}(\gamma^{2k}-1)\right].$$

Substituting equation (20) into equation (16),

$$f(\gamma,z,t)=\gamma\frac{\partial R}{\partial z}w-\frac{R}{\gamma}\int_0^{\gamma}\gamma\left[-\frac{2w}{R}\frac{\partial R}{\partial z}+\frac{2}{R}\frac{\partial R}{\partial t}\Big(\frac{1}{N}\sum_{k=1}^{N}\frac{k+1}{k}(\gamma^{2k}-1)\right]d\gamma-\frac{2}{\gamma}\frac{\partial R}{\partial z}\int_0^{\gamma}w\gamma d\gamma\,,$$

$$f(\gamma,z,t)=\gamma\frac{\partial R}{\partial z}w+\frac{R}{\gamma}\int_0^{\gamma}\gamma\frac{2w}{R}\frac{\partial R}{\partial z}d\gamma-\frac{R}{\gamma}\int_0^{\gamma}\frac{2\gamma}{R}\frac{\partial R}{\partial t}\frac{1}{N}\sum_{k=1}^{N}\frac{k+1}{k}(\gamma^{2k}-1)d\gamma-\frac{2}{\gamma}\frac{\partial R}{\partial z}\int_0^{\gamma}w\gamma d\gamma,$$

$$f(\gamma,z,t)=\gamma\frac{\partial R}{\partial z}w+\frac{2}{\gamma}\frac{\partial R}{\partial z}\int_0^{\gamma}w\gamma d\gamma-\frac{2}{\gamma N}\frac{\partial R}{\partial t}\sum_{k=1}^{N}\frac{k+1}{k}\int_0^{\gamma}(\gamma^{2k+1}-\gamma)d\gamma-\frac{2}{\gamma}\frac{\partial R}{\partial z}\int_0^{\gamma}w\gamma d\gamma\,,$$

(21) $$f(\gamma,z,t)=\gamma\frac{\partial R}{\partial z}w-\frac{2}{\gamma N}\frac{\partial R}{\partial t}\sum_{k=1}^{N}\frac{k+1}{k}\int_{0}^{\gamma}(\gamma^{2k+1}-\gamma)d\gamma\,.$$

Integrating and expanding equation (21), the velocity profile in the radial direction is

(22) $$f(\gamma,z,t)=\gamma\frac{\partial R}{\partial z}w+\gamma\frac{\partial R}{\partial t}-\frac{\gamma}{N}\frac{\partial R}{\partial t}\sum_{k=1}^{N}\frac{1}{k}\left(\gamma^{2k}-1\right).$$

Assume the velocity profile in the axial direction is

(23) $$w(\gamma,z,t)=\sum_{k=1}^{N}q_k(\gamma^{2k}-1)\,.$$

Choose $N=1$,

(24) $$f(\gamma,z,t)=\gamma\frac{\partial R}{\partial z}w+\gamma\frac{\partial R}{\partial t}-\frac{\partial R}{\partial t}\gamma(\gamma^{2k}-1)\,,$$

(25) $$w(\gamma,z,t)=q(z,t)(\gamma^{2}-1)\,.$$

Using the equation of axil and radial velocity profile, radial coordinate and the continuity equation Plugging equations (24) and (25) into equations (13) and (14), the Navier-Stokes equations get the forms as below to determine the variable $q(z,t)$ and $R(z,t)$.

(26) $$\frac{\partial q}{\partial t}-\frac{4q}{R}\frac{\partial R}{\partial t}-\frac{2q^2}{R}\frac{\partial R}{\partial z}+\frac{4\mu}{R^2}+\frac{1}{\rho}\frac{\partial p}{\partial z}=0\,,$$

(27) $$2\frac{\partial R}{\partial t}+\frac{R}{2}\frac{\partial q}{\partial z}+q\frac{\partial R}{\partial z}=0\,.$$

Multiplying equation (27) with π,

(28) $$2\pi\frac{\partial R}{\partial t}+\frac{\pi R}{2}\frac{\partial q}{\partial z}+\pi q\frac{\partial R}{\partial z}=0\,.$$

The cross-sectional area of the blood vessel is

(29) $$S=\pi R^2.$$

The blood flow rate is given by

(30) $$Q=\int\int w\partial\gamma=\frac{1}{2}q\pi R^2\,.$$

By using the chain rule, $\frac{\partial Q}{\partial z}$ and $\frac{\partial S}{\partial t}$ are obtained as:

$$\frac{\partial Q}{\partial z}=\pi qR\frac{\partial R}{\partial z}+\frac{\pi R^2}{2}\frac{\partial q}{\partial z}\,,$$

$$\frac{\partial S}{\partial t}=2\pi R\frac{\partial R}{\partial t}\,.$$

Substituting in equation (28),

$$\frac{1}{R}\left(\frac{\partial S}{\partial t}+\frac{\partial Q}{\partial z}\right)=0\,,$$

(31) $$\frac{\partial S}{\partial t}+\frac{\partial Q}{\partial z}=0\,.$$

Inserting the value of $\frac{\partial q}{\partial t},\frac{\partial q}{\partial z},\frac{\partial R}{\partial t},\frac{\partial R}{\partial z}$ in equation (12), another differential equation is obtained.

(32) $$\frac{\partial Q}{\partial t}+\frac{2Q}{S}\frac{\partial Q}{\partial z}-\frac{2Q^2}{S^2}\frac{\partial S}{\partial z}+\frac{4\pi\mu}{S}Q+\frac{S}{2\rho}\frac{\partial p}{\partial z}=0,\ \text{where } \frac{\partial Q}{\partial z}=-\frac{\partial S}{\partial t}.$$

This equation is called the cardiovascular system equation.

3. Modeling of the blood flow rate

The cross-section area of the blood vessel remains unchanged with time and constant over distance. The pressure gradient is also constant over the distance.

Thus $\frac{\partial S}{\partial t}=0$ and $\frac{\partial S}{\partial z}=0.$

The cardiovascular system equation becomes,

(33) $$\frac{\partial Q}{\partial t}+\frac{4\pi\mu}{S}Q+\frac{S}{2\rho}\frac{\partial p}{\partial z}=0.$$

This is the one - dimensional mathematical model of the blood flow rate.

4 The flow rate of the blood flow

Let 'μ' be a liquid of coefficient of viscosity flow in streamline motion through a horizontal tube of radius R and length L.

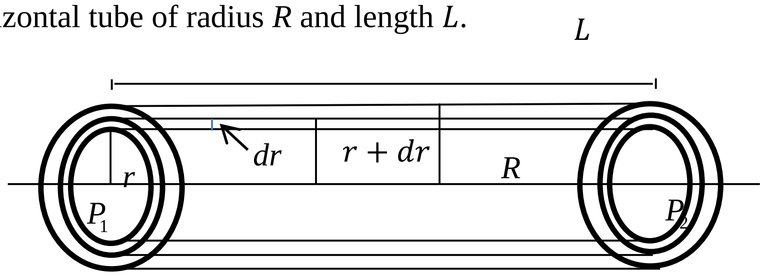

Figure 1

The resistance force is given by

$$F_R=\mu A\frac{dv}{dr}\,,\ (A = \text{area of the imaginary cylinder})$$

(34) $$F_R=\mu 2\pi rL\frac{dv}{dr}$$

and if P is the pressure difference between the two ends, then force (F_a) accelerating the liquid is given by

$F_a = P \ A$, (A= area of the opening of the imaginary)

(35) $F_a = P \ \pi r^2$.

For steady flow of liquid,

$$F_R = -F_a$$

$$\mu L 2\pi r \frac{dv}{dr} = -P\pi r^2$$

$$dv = \frac{-P}{2\mu l} rdr \ .$$

By integrating,

(36) $v = \frac{-P}{2\mu L} \frac{r^2}{2} + C_1$.

At $v = 0, r = R,$ so equation (36) can be written as

$$0 = \frac{-P}{2\mu L} \frac{R^2}{2} + C_1$$

$$C_1 = \frac{-PR^2}{4\mu l}.$$

Substituting the value of C_1 in equation (36),

$$v = \frac{-P}{2\mu l} \frac{r^2}{2} + \frac{PR^2}{4\mu l} \ ,$$

(37) $v = \frac{-P}{4\mu l}(R^2 - r^2)$.

Then imagine co-axial tube of radius $r + dr$, enclosing the imaginary tube of radius r.

Thus, the area between the two tubes is $2\pi rdr$.

So, volumetric of liquid passing through the area is

$Q = A.v \quad = 2\pi rdr.v.$

So, volumetric of liquid (Q) passing forward through whole tube is

$$Q = \int_0^R 2\pi rvdr$$

$$= \int_0^R 2\pi r \frac{P}{4\mu l}(R^2 - r^2)dr$$

$$= \frac{1}{2} \frac{\pi P}{\mu l}\left[\frac{2R^4 - R^4}{4}\right],$$

(38) $Q = \frac{\pi PR^4}{8\mu L}$.

This equation is the flow rate of the blood flow. The volumetric flow rate is directly proportional to the pressure gradient and radius. It is inversely proportional to the viscosity of the blood. Thus, a small change in channel radius can causes a small change in flow rate and a small change in radius can cause a small in pressure gradient. An effective way of changing blood pressure is change in vessel radius.

3.1Example

The blood flow through a large artery of radius 2.5 *mm* is found to be 20 *cm* long. The pressure across the artery ends is 380 *Pa*, the rate of the blood flow can be calculated.

The blood viscosity $\mu = 0.0027 Ns/m^2$

Radius of a large artery $R = 2.5\ mm = 2.5(10^{-3})\ m$

The length of the artery $L = 20\ cm = 0.20\ mm$

The difference of pressure = 380 *Pa*

The rate of blood flow is

$$Q = \frac{\pi P R^4}{8\mu L} = 1.0789(10^{-5})\ m^3/s.$$

4. Modeling of the blood pressure

The relation between blood flow rate and the pressure is given by

$$Q = \frac{\pi R^4 P}{8\mu L},$$

where L is the length, P is pressure gradient and R is the radius of vessel.

Substituting in equation (33),

$$\frac{\pi R^4}{8\mu L}\frac{dP}{dt} + \frac{4\pi\mu}{S}\frac{\pi R^4}{8L\mu}P + \frac{S}{2S}\frac{\partial P}{\partial z} = 0.$$

$$(39) \qquad \frac{dP}{dt} + \frac{4\mu}{R^2}P + \frac{4\mu L}{\rho R^2}\frac{\partial P}{\partial z} = 0.$$

This equation is the mathematical model of the blood pressure in the body.

4.1 Example

Blood is flowing through an artery of radius 2 *mm* at a rate of 40 *cm/s*. The flow rate and the volume that passes through the artery in a period of 1*s* can be determined.

The radius of an artery $r = 2\ mm = 0.2\ cm.$

The speed of the blood flow $v = 40\ cm/s.$

The volumetric flow rate $Q = Av$

$$Q = 3.14(0.2)^2 40$$
$$Q = 5.024 cm^3/s$$

The volume passes through the artery in 1*s* $= Qt = 5.024\ cm^3$.

4.2 Example

The heart of a resting adult pumps blood at a rate of 5 liter per minute. The rate of volumetric flow per second can be evaluated.

The blood flow rate $Q \quad = 5.00\ L/\ min$

To calculate the volumetric flow rate in centimeter cube per second,

$$5 \text{ liter} = 5000\ cm^3$$

$$Q \quad = \frac{5000}{60} = 83.3\ cm^3/s.$$

To evaluate the volumetric flow rate in meter cube per second,

$$5 \text{ liters} \quad = \frac{5}{1000} m$$

$$Q \quad = \frac{5}{1000 \times 60} \quad = 83.3{\times}10^{-6}\ m^3/s.$$

Conclusion

The goal of this paper was to represent a mathematical model of the blood flow. This model is able to show that the blood flow rate is influenced by the change of cross-sectional area and the pressure gradient. The model can also show that the blood pressure is influenced by both of the cross-sectional area and the length of the blood vessel.

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